Sliding Window Tone Reservation Technique for the Peak-to-Average Power Ratio Reduction of FBMC-OQAM Signals

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Abstract—The filter bank multicarrier with offset quadrature amplitude modulation (FBMC-OQAM) has attracted increasing attention recently. In this paper, we address the problem of PAPR reduction for FBMC-OQAM systems using Tone Reservation (TR) technique. Due to the overlapping structure of FBMC-OQAM signals, directly applying TR schemes of OFDM systems to FBMC-OQAM systems is not effective. We improve the tone reservation (TR) technique by employing sliding window for the PAPR reduction of FBMC-OQAM signals, called sliding window tone reservation (SW-TR) technique. Furthermore, we propose a method to overlap the sliding windows to control the out-of-window peak regrowth caused by the peak-canceling signal.

Index Terms—FBMC-OQAM, peak-to-average power ratio, sliding window, tone reservation.

I. INTRODUCTION

The filter bank multicarrier with offset quadrature amplitude modulation (FBMC-OQAM) has attracted increasing attention recently, due to its high spectrum efficiency, low sidelobes and robustness to the narrow-band interference [1]-[4]. Similar to other multicarrier systems like OFDM, a fundamental drawback of FBMC-OQAM systems is the high peak-to-average power ratio (PAPR) of the signal, which degrades the efficiency of a high power amplifier. Over the past decade, various PAPR reduction techniques for OFDM have been proposed, among which tone reservation (TR) [5]-[8] attracted much attention. The TR technique is simple, effective and it causes no interference to the data signal. Due to the similarity between the FBMC-OQAM and OFDM systems, it is natural to consider employing TR to reduce the PAPR of FBMC-OQAM systems. However, FBMC-OQAM signals have a very different signal structure compared with the OFDM signals: signals of adjacent data blocks overlap with each other for FBMC-OQAM systems, while they are independent for OFDM systems. Therefore, directly applying TR schemes of OFDM systems to FBMC-OQAM systems is not effective.

In this paper, we propose a sliding window tone reservation (SW-TR) technique for the PAPR reduction of FBMC-OQAM signals, called SW-TR technique. Moreover, we propose a method to overlap the sliding windows to control the out-of-window peak regrowth caused by the peak-canceling signal.

II. FBMC-OQAM SIGNAL MODEL

At the transmitter of a typical FBMC-OQAM system [9], the complex input symbols are written as

$$X^n_m = a^n_m + j \times b^n_m, \quad 0 \leq n \leq N - 1, \quad 0 \leq m < \infty,$$  

(1)

where $N$ is a positive integer. $a^n_m$ and $b^n_m$ are the real and imaginary parts of the $m$'th symbol on the $n$'th tone, respectively. The $m$'th symbols on all tones form a data block $X_m = [X^0_m, X^1_m, \ldots, X^{N-1}_m]^T$. The in-phase and quadrature components are staggered in time domain by $T/2$, where $T$ is the symbol period. Then, the symbols are passed through a bank of transmission filters and modulated using $N$ tone modulators whose carrier frequencies are $1/T$-spaced apart.

Generally speaking, we are more concerned with the reduction of the PAPR of the continuous-time FBMC-OQAM signal. However, most existing PAPR reduction schemes can only be implemented for discrete-time signals. To approximate the true PAPR of the signal, the FBMC-OQAM signal is sampled with sampling period $T/F$, where $F = LN$ and $L$ is the oversampling factor. It was known in [10] that the PAPR of the sampled signal approximates to the true PAPR of the continuous-time signal very well for OFDM signals when $L \geq 4$.

The discrete time domain signal of symbol $X^n_m$ can be expressed with the following sequence.

$$x^n_m[k] = \left\{ \begin{array}{ll}
[a^n_m \times h(k - mF) + j \times b^n_m \times h(k - mF - \frac{F}{2})] \\
\times e^{j(n-1)(\frac{2\pi}{T}k + \frac{\pi}{4})}, & mF \leq k \leq (m + A + 1)F - 1 \\
0, & \text{else}
\end{array} \right.$$

(2)

where $h[k]$ is the discrete-time filter obtained by sampling the continuous-time filter $h(t)$, and $A$ represents the number of succeeding data blocks that are overlapped with $x^n_m[k]$ (and it could be determined by the length of filter $h[k]$). Similar to the OFDM signal, the time domain FBMC-OQAM signal of the $n$'th data block is a superposition of the $N$ tones, i.e.,

$$x_m[k] = \sum_{n=0}^{N-1} x^n_m[k], \quad -\infty < k < \infty.$$  

(3)

Different from the non-overlapping OFDM data blocks, the time domain FBMC-OQAM signals of adjacent data blocks
overlap with each other. The time domain signal of all the \( M \) data blocks at the output of the transmitter can be written as

\[
x[k] = \sum_{m=0}^{M-1} x_m[k], \quad 0 \leq k < \infty.
\] (4)

In TR schemes, a small number of tones are reserved as a PRTs. Denote the ordered set of indices of the reserved tones by \( R = \{r_0, r_1, ..., r_{Z-1}\} \), where \( Z \) is the size of the PRT set. With TR schemes, the \( m \)th data block \( x_m \) consists of two parts, the peak reduction signals on the PRTs and the data signals on the unreserved tones, respectively, i.e.,

\[
X_m^n = C_m^n + \hat{X}_m^n = \begin{cases} 
C_m^n, & n \in R \\
\hat{X}_m^n, & n \in R^C 
\end{cases}
\] (5)

where \( R^C \) is the complement set of \( R \) in \( R = \{0, 1, ..., N-1\} \), and \( C_m^n \) and \( \hat{X}_m^n \) represent the peak reduction and data signal on the \( n \)th tone of the \( m \)th data block, respectively, and

\[
C_m^n = 0, \quad \text{for } n \in R^C, \quad \hat{X}_m^n = 0, \quad \text{for } n \in R. \quad (6)
\]

A TR scheme selects proper \( C_m \) so that the peak power of the signal is greatly reduced, where \( C_m = \{C_m^0, C_m^1, ..., C_m^{N-1}\} \). Since the OFDM signals of the adjacent data blocks do not overlap, all TR schemes for OFDM signals independently determines \( C_m \) for each data block \( m \). Therefore, directly applying the TR schemes of OFDM systems to FBMC-OQAM systems is thus ineffective due to the reason that signals of adjacent FBMC-OQAM data blocks overlap with each other.

III. SLIDING WINDOW TR TECHNIQUE FOR THE FBMC-OQAM SYSTEM

In this section, we propose a sliding window TR technique to reduce the PAPR of the FBMC-OQAM signal. Moreover, to control the peak regrowth caused by the peak-canceling signal, we propose the overlapping sliding window scheme to help improve the PAPR reduction.

A. Sliding Window Tone Reservation (SW-TR)

The basic procedure of the proposed SW-TR technique is: firstly use the PRTs of several consecutive data blocks to cancel the peaks of the FBMC-OQAM signal inside a window, and then slide the window when the threshold of peak or the maximum number of iterations is reached.

In this paper, we denote the length of the window by \( W = w \times F \), where \( w \) is an integer that could be used to adjust the length of the window. Due to the overlapping structure of the FBMC-OQAM signal, there are \( P \) adjacent FBMC-OQAM data blocks overlapping with one sliding window in time domain, while the \( P \) data blocks are denoted as the \( G(l) \)th data block, \((G(l)+1)\)th data block,..., \((G(l)+P-1)\)th data block, respectively. \( G(l) \) and \( P \) can be readily determined when \( A \) and \( w \) are given. Obviously, \( C_{G(l)}, C_{G(l)+1}, ..., C_{G(l)+P-1} \) contribute to the peak reduction of the signal in the \( l \)th window. The SW-TR algorithm presented below determines \( C_{G(l)}, C_{G(l)+1}, ..., C_{G(l)+P-1} \) for the \( l \)th window, and slide the window to the next position.

Step 1: After the \((l-1)\)th window is well processed, extract the signal in the \( l \)th window from \( x[k] \) (Fig.1(a)) as shown in Fig. 1(b), and denote it by the following sequence

\[
w_l[k] = \begin{cases} 
x[k], & (l-1)\times W \leq k \leq lW - 1 \\
0, & \text{else}
\end{cases}
\] (7)

Step 2: Similar to [10], the objective is to clip the amplitude of the signal with a predefined threshold \( B \) (shown in Fig. 1(b))

\[
\tilde{w}_{l}[k] = \begin{cases} 
w_l[k], & \mid w_l[k] \mid \leq B \\
Be^{\angle w_l[k]}, & \mid w_l[k] \mid > B
\end{cases}
\] (8)

where \( \angle w_l[k] \) is the phase of \( w_l[k] \). The threshold \( B \) affects the PAPR reduction performance and is always obtained from simulation results. The expected clipping signal corresponding to (8) is \( f_l[k] = \tilde{w}_{l}[k] - w_l[k] \), i.e.,

\[
f_l[k] = \begin{cases} 
0, & \mid w_l[k] \mid \leq B \\
(B - \mid w_l[k] \mid)e^{\angle w_l[k]}, & \mid w_l[k] \mid > B
\end{cases}
\] (9)

Though \( f_l[k] \) can cancel the peak of the signal to the predefined threshold, it brings interference to the data tones and degrades the bit error rate performance of the system. Therefore, it is better to approximate the clipping signal \( f_l[k] \).
to \( \hat{f}_l[k] \), which only has nonzero signal on the reserved tones. Similar to [11], it needs several iterations to obtain \( C_{G(l)}, C_{G(l)+1}, \ldots, C_{G(l)+P-1} \) that produce \( \hat{f}_l[k] \). The iteration is quite the same as that in the OFDM system [11], the only difference is that the TR technique of the FBMC-OQAM system utilizes the reserved tones of several data blocks rather than those of a single data block. If the maximum number of iterations is reached or the threshold level \( B \) is obtained, the iteration stops.

Then, the peak-canceling signal is

\[
\hat{f}_l[k] = \begin{cases} 
G(l)+P-1 \sum_{m=G(l)} \ c_m[k], & (G(l) - 1)F < k \\
0, & \text{else}
\end{cases},
\]

(10)

where \( c_m[k] \) is the time domain sequence corresponding to \( C_m \).

Step 3: replace \( x[k] \) with

\[
x[k] \leftarrow x[k] + \hat{f}_l[k].
\]

Step 4: Slide the window, i.e., let \( l \leftarrow l + 1 \), and go to Step 1.

Since the proposed SW-TR scheme uses the peak reduction tones of several consecutive data blocks to cancel the peaks of the FBMC-OQAM signal inside a window, compared with traditional TR scheme, an extra processing delay of \( AT - 1 \) is incurred, where \( AT - 1 \) is length of the filter and \( T \) is the symbol interval.

**B. Overlapping Sliding Window to Control the Out-of-Window Peak Regrowth**

Obviously, the length of the nonzero part of \( \hat{f}_l[k] \) is larger than that of the sliding window as shown in Fig. 1(c). The out-of-window part of \( \hat{f}_l[k] \) can be represented by

\[
\Phi_l[k] = \begin{cases} 
\hat{f}_l[k], & -\infty < k \leq (l-1)W - 1 \text{ or } lW \leq k \leq \infty \\
0, & \text{else}
\end{cases}.
\]

(12)

The part of \( \Phi_l[k] \) before the \( l \)th window,

\[
\Phi_l^w[k] = \begin{cases} 
\hat{f}_l[k], & -\infty < k \leq (l-1)W - 1 \\
0, & \text{else}
\end{cases},
\]

(13)

overlaps with the signal in the previous window, which could cause regrown peaks and damage the performance of the proposed scheme. On the other hand, the part of \( \hat{f}_l[k] \) after the window is not a serious issue, since it can be handled by the clipping of the succeeding windows.

In this subsection, we propose an overlapping sliding window scheme to control the pre-window peak regrowth, which overlaps the adjacent sliding windows. The length of the overlapping part of two adjacent windows is denoted as \( V \).

In other words, the starting points of two adjacent windows have an offset of \( O \), where \( O = W - V \), which is shown in Fig. 1(d). With the overlapping sliding windows, (7) in the SW-TR algorithm is replaced with

\[
w_l[k] = \begin{cases} 
x[k], & (l-1)O \leq k \leq (l-1)O + W - 1 \\
0, & \text{else}
\end{cases}.
\]

(14)

The other steps of the algorithm do not need to be changed.

The reason that the overlapping sliding window scheme can help to control the out-of-window regrowth could be explained as follows. After the clipping of the \((l-1)\)th window, the peak signal in the overlapping part of the \((l-1)\)th and \( l \)th window is already lower than or approximates to the threshold \( B \). In other words, the peaks to be clipped in the \( l \)th window are mainly distributed over the part that is not overlapped with the \((l-1)\)th window (the part on the right hand side of the \( l \)th window as shown in Fig. 1(c)). Consequently, the energy of the peak-canceling signal \( \hat{f}_l[k] \) is mainly distributed in the right hand side of the \( l \)th window and post-window section. Therefore, the energy of \( \Phi_l^w[k] \) is smaller than that of the method without overlapped window, which leads to less pre-window peak regrowth.

**IV. Simulation Results**

In this section, simulations are conducted to investigate the PAPR reduction performance of the proposed SW-TR technique. The FBMC-OQAM system employs 64 tones, where 56 tones are used for data and eight tones are reserved as PRTs, i.e., \( Z = 8 \). All data tones are QPSK modulated. The oversampling factor is \( L = 1 \). The PRT tones are selected randomly, since it is known that randomly generated PRT sets perform better in PAPR reduction than contiguous PRT sets and interleaved PRT sets on average [12]. The number of the iterations for the TR schemes is 50 in the simulations. The length of the prototype filter \( h[k] \) is chosen to be \( 4F - 1 \), i.e., its time duration is about four times of \( T \). Thus, a FBMC-OQAM data block overlaps with four succeeding data blocks, i.e., \( A = 4 \). Complementary cumulative distribution function (CCDF) is employed as measurement of PAPR reduction performance in the simulations.

Fig. 2 shows the PAPR performance of the proposed SW-TR technique with different thresholds. The curve “FBMC-OQAM original PAPR” represents the performance of the FBMC-OQAM system without TR. The length of the sliding window is \( W = 3F \), and the length of the overlapping part is set as \( V = 2F \). We normalize threshold \( B \) with
the power of the signal, and the normalized threshold is denoted as $R = B / \sqrt{E[|x|^2]}$. It is observed that the PAPR performance of the SW-TR technique varies with different $R$. Among all simulated thresholds, the best PAPR reduction performance in the CCDF range from 0.01 to 0.001 is achieved with $R = 1.9(5.58\text{dB})$. Therefore, we fixed $R = 5.58\text{dB}$ in the following simulations for the SW-TR technique.

In simulation of Fig. 3, the length of the overlapping part of two adjacent sliding windows, $V$, is varied. The length of the sliding window is $W = 3F$, and the length of the overlapping part is set as $V = 2F$, $V = F$ and $V = 0$, respectively, where $V = 0$ means sliding windows without overlapping. As shown in Fig. 3, the corresponding PAPR at CCDF of $10^{-4}$ is 6.8dB, 7.1dB and 8.0dB, respectively. Therefore, it is concluded that overlapping sliding windows is quite effective for the proposed SW-TR technique.

Fig. 4 compares the PAPR reduction of the proposed SW-TR technique with different $W$. The length of the overlapping part is set as $V = W - F$. It is obvious that the best performance is achieved with the largest $W$, $W = 4F$ and $V = 3F$ among all simulated parameters. Compared with the PAPR performance of the conventional TR method for the FMBC-OQAM system, the SW-TR method outperforms almost 2dB at $CCDF = 0.0001$, with $W = 4F$ and $V = 3F$. In addition, we compare the PAPR performance of the following two systems: 1) the FMBC-OQAM system with the proposed SW-TR technique; 2) an OFDM system with TR technique, which has the same subcarrier number, PRT tone number, and modulation type with those of the FMBC-OQAM system. As shown in the simulation results, we can draw the conclusion that when the length of the window $W$ is large enough, the PAPR reduction performance of the proposed SW-TR method for the FMBC-OQAM system is even better than that of the TR method for the OFDM system.

V. CONCLUSION

Due to the signal structure difference between FMBC-OQAM and OFDM systems, the TR technique for OFDM systems is not suitable for the FMBC-OQAM system. In this paper, a SW-TR technique has been proposed for the PAPR reduction of FMBC-OQAM systems. The simulation results showed that the proposed SW-TR technique is effective in reducing the PAPR of the FMBC-OQAM signal.

REFERENCES